1a) $D = \mathbb{R}\backslash\{0\}$

$$\frac{(2-h)^3-(2-h)^2-4}{-h} = \frac{8-12h+6h^2-h^3-4+4h-h^2-4}{-h} = \frac{-h^3+5h^2-8h}{-h} = \frac{-h\left(h^2-5h+8\right)}{-h} =$$
$$= \underline{\underline{h^2-5h+8}}$$

1b) $D = \mathbb{R}\backslash\{-2;\ 2\}$

$$\frac{9x^2-63x+90}{3x^2-12} = \frac{9\left(x^2-7x+10\right)}{3\left(x^2-4\right)} = \frac{3(x-2)(x-5)}{(x-2)(x+2)} = \underline{\underline{\frac{3(x-5)}{x+2}}}$$

1c) $D = \mathbb{R}\backslash\{0\}$

$$\frac{\frac{1}{2}(-2+t)^2-(-2+t)^3-10}{t} = \frac{\frac{1}{2}\left(4-4t+t^2\right)-\left(-8+12t-6t^2+t^3\right)-10}{t} =$$
$$= \frac{2-2t+\frac{1}{2}t^2+8-12t+6t^2-t^3-10}{t} = \frac{-14t+6{,}5t^2-t^3}{t} = \underline{\underline{-14+6{,}5t-t^2}}$$

1d) $D = \mathbb{R}\backslash\{-5;\ 3\}$

$$\frac{12x^2-108}{4x^2+8x-60} = \frac{12\left(x^2-9\right)}{4\left(x^2+2x-15\right)} = \frac{3(x-3)(x+3)}{(x+5)(x-3)} = \underline{\underline{\frac{3(x+3)}{x+5}}}$$

2 a) $\frac{2x}{x^2-9} - \frac{1}{x^2+3x} = \frac{2}{x-3}$

$\frac{2x}{(x+3)(x-3)} - \frac{1}{x(x+3)} = \frac{2}{x-3}$ $\quad |\cdot \text{HN}$, wobei $\text{HN} = x(x+3)(x-3)$

$D = \mathbb{R}\backslash\{-3;\ 0;\ 3\}$

$$2x^2-(x-3) = 2x(x+3)$$
$$2x^2-x+3 = 2x^2+6x$$
$$7x = 3$$
$$x = \frac{3}{7} \qquad \underline{\underline{L = \left\{\frac{3}{7}\right\}}}$$

2 b) $\sqrt{4x+25} = x-5 \quad |^2$ $\qquad$ Bedingung für D: $4x+25 \geq 0$

$x \geq -\frac{25}{4}$

$\underline{\underline{D = [-6{,}25;+\infty[}}$

$$4x+25 = x^2-10x+25$$
$$x^2-14x = 0$$
$$x(x-14) = 0$$

$x_1 = 0$ $\quad$ Probe für $x_1: \sqrt{25} = -5 \quad (f) \quad \Rightarrow x_1 \notin L$

$x_2 = 14$ $\quad$ Probe für $x_2: \sqrt{25} = 5 \quad (w) \quad \Rightarrow x_2 \in L$

$\Rightarrow \underline{\underline{L = \{14\}}}$

2 c) $\frac{4}{x^2-4} + \frac{3x}{x+2} = \frac{9x-2}{x-2}$ $\quad |\cdot \text{HN}$, wobei $\text{HN} = (x+2)(x-2) = x^2-4$

$D = \mathbb{R}\backslash\{-2;\ 2\}$

$$4+3x(x-2) = (9x-2)(x+2)$$
$$4+3x^2-6x = 9x^2+18x-2x-4$$
$$-6x^2-22x+8 = 0$$
$$3x^2+11x-4 = 0$$
$$x_{1/2} = \frac{-11\pm\sqrt{121+48}}{6} = \frac{-11\pm 13}{6}$$

$x_1 = \frac{1}{3} \qquad x_2 = -4 \qquad \underline{\underline{L = \left\{-4;\frac{1}{3}\right\}}}$

2 d) $\sqrt{x}-2=\sqrt{x-1} \quad |^2$ Bedingung für D: $x \geq 0 \wedge x-1 \geq 0$

$x-4\sqrt{x}+4=x-1$ $x \geq 0 \wedge x \geq 1$

$-4\sqrt{x}=-5$ $x \geq 1$

$16x=25$ $\underline{\underline{D=[1;+\infty[}}$

$x_0=\frac{25}{16}$

Probe für x_0: $\frac{5}{4}-2=\frac{3}{4}$ (f) $\Rightarrow x_0 \notin L$

$\Rightarrow \underline{\underline{L=\{\ \}}}$

3.0 $\frac{1}{4}x^2+kx+5=0$; $x \in \mathbb{R}; k \in \mathbb{R}$.

3.1 $k=7$: $\frac{1}{4}x^2+7x+5=0$

$$x_{1/2}=\frac{-7\pm\sqrt{49-5}}{\frac{1}{2}}=2\cdot\left(-7\pm 2\sqrt{11}\right)=-14\pm 4\sqrt{11}$$

$$\underline{\underline{L_7=\left\{-14-4\sqrt{11};-14+4\sqrt{11}\right\}}}$$

3.2 Diskriminante der quadratischen Gleichung: $D=k^2-4\cdot\frac{1}{4}\cdot 5=k^2-5$.

Die quadratische Gleichung hat <u>keine reelle Lösung</u>

$\Leftrightarrow \quad D<0 \quad \Leftrightarrow \quad k^2-5<0 \quad \Leftrightarrow \quad k^2<5 \quad \Leftrightarrow \quad |k|<\sqrt{5} \quad \Leftrightarrow \quad \underline{\underline{-\sqrt{5}<k<\sqrt{5}}}$

4.0 $x^2-\frac{1}{2}x+b=0$; $x \in \mathbb{R}; b \in \mathbb{R}$.

4.1 $b=-3$: $x^2-\frac{1}{2}x-3=0$

$$x_{1/2}=\frac{\frac{1}{2}\pm\sqrt{\frac{1}{4}+12}}{2}=\frac{\frac{1}{2}\pm 3{,}5}{2}$$

$x_1=2 \quad x_2=-1{,}5$

$\underline{\underline{L_{-3}=\{-1{,}5;2\}}}$

4.2 Diskriminante der quadratischen Gleichung: $D=\frac{1}{4}-4b$.

Die quadratische Gleichung besitzt <u>zwei verschiedene reelle Lösungen</u>

$\Leftrightarrow \quad D>0 \quad \Leftrightarrow \quad \frac{1}{4}-4b>0 \quad \Leftrightarrow \quad 4b<\frac{1}{4} \quad \Leftrightarrow \quad \underline{\underline{b<\frac{1}{16}}}$

5.0 $ax+2x-8<7x+4$; $x \in \mathbb{R}; a \in \mathbb{R}$.

5.1 $a=2$

$2x+2x-8<7x+4$

$-3x<12 \quad |:(-3)$

$x>-4$

$\underline{\underline{L_2=]-4;+\infty[}}$

5.2 $ax + 2x - 8 < 7x + 4$

$ax - 5x < 12$

$(a-5)x < 12 \quad |:(a-5), \quad \text{falls} \quad a - 5 \neq 0$

1.Fall: $a - 5 > 0 \quad \Leftrightarrow \quad a > 5$

$x < \frac{12}{a-5}$

$L_a = \left]-\infty; \frac{12}{a-5}\right[$

2.Fall: $a - 5 < 0 \quad \Leftrightarrow \quad a < 5$

$x > \frac{12}{a-5}$

$L_a = \left]\frac{12}{a-5}; +\infty\right[$

3.Fall: $a - 5 = 0 \quad \Leftrightarrow \quad a = 5$

$0 \cdot x < 12$ allgemeingültig für alle $x \in \mathbb{R}$

$L_a = L_5 = \mathbb{R}$

6.0 $ax - 5x + 6 > 11 - 3x \quad ; \quad x \in \mathbb{R}; a \in \mathbb{R}.$

6.1 $a = -5$

$-5x - 5x + 6 > 11 - 3x$

$-7x > 5 \quad |:(-7)$

$x < -\frac{5}{7}$

$L_{-5} = \left]-\infty; -\frac{5}{7}\right[$

6.2 $ax - 5x + 6 > 11 - 3x$

$ax - 2x > 5$

$(a-2)x > 5 \quad |:(a-2), \quad \text{falls} \quad a - 2 \neq 0$

1.Fall: $a - 2 > 0 \quad \Leftrightarrow \quad a > 2$

$x > \frac{5}{a-2}$

$L_a = \left]\frac{5}{a-2}; +\infty\right[$

2.Fall: $a - 2 < 0 \quad \Leftrightarrow \quad a < 2$

$x < \frac{5}{a-2}$

$L_a = \left]-\infty; \frac{5}{a-2}\right[$

3.Fall: $a - 2 = 0 \quad \Leftrightarrow \quad a = 2$

$0 \cdot x > 5$ unerfüllbar für alle $x \in \mathbb{R}$

$L_a = L_2 = \{\ \}$

7 a) $|x + 5| \geq 13$

$x + 5 \leq -13 \quad \vee \quad x + 5 \geq 13$

$x \leq -18 \quad \vee \quad x \geq 8$

$L =]-\infty; -18] \cup [8; +\infty[$

7 b) $x^2 - 10x + 21 < 0$

$x^2 - 10x + 25 < -21 + 25$

$(x-5)^2 < 4$

$|x - 5| < 2$

$-2 < x - 5 < 2$

$3 < x < 7$

$L =]3; 7[$

7 c) $x^2 + 6x + 4 > 0$

$x^2 + 6x + 9 > -4 + 9$

$(x+3) > 5$

$|x + 3| > \sqrt{5}$

$x + 3 < -\sqrt{5} \quad \vee \quad x + 3 > \sqrt{5}$

$x < -3 - \sqrt{5} \quad \vee \quad x > -3 + \sqrt{5}$

$L =]-\infty; -3 - \sqrt{5}[\cup]-3 + \sqrt{5}; +\infty[$

7 d) $\frac{5x+8}{3x+4} \le 0$

$(5x+8 \ge 0 \wedge 3x+4<0) \vee (5x+8 \le 0 \wedge 3x+4>0)$

$\left(x \ge -\frac{8}{5} \wedge x < -\frac{4}{3}\right) \vee \left(x \le -\frac{8}{5} \wedge x > -\frac{4}{3}\right)$

$(x \ge -1{,}6 \wedge x < -1{,}\overline{3}) \vee (x \le -1{,}6 \wedge x > -1{,}\overline{3})$

unerfüllbar in $\mathbb{R}$

$-1{,}6 \le x < -1{,}\overline{3}$

$L = \left[-\frac{8}{5}; -\frac{4}{3}\right[$

8 a) $x^6 = 25$

$x_{1/2} = \pm\sqrt[6]{25}$

$x_{1/2} = \pm\sqrt[3]{5}$

$x_{1/2} \approx \pm 1{,}710$

8 b) $x^3 = -7$

$x = -\sqrt[3]{7}$

$x \approx -1{,}913$

8 c) $5^x = 3$

$x = \log_5 3$

$x = \frac{\ln 3}{\ln 5}$

$x \approx 0{,}683$

8 d) $13^x = 39$

$x = \log_{13} 39$

$x = \frac{\ln 39}{\ln 13}$

$x \approx 1{,}428$

8 e) $3^x = 27\sqrt{3}$

$3^x = 3^3 \cdot 3^{\frac{1}{2}}$

$3^x = 3^{3,5}$

$x = 3{,}5$

8 f) $5^{-x} = \frac{\sqrt{5}}{125}$

$5^{-x} = \frac{5^{\frac{1}{2}}}{5^3}$

$5^{-x} = 5^{-2,5}$

$-x = -2{,}5$

$x = 2{,}5$

8 g) $7^{2x} - 2 \cdot 7^x = 15$

Substitution: $u := 7^x$

$u^2 - 2u - 15 = 0$

$(u-5)(u+3) = 0$

$u = 5 \vee u = -3$

Rücksubstitution: $7^x = 5 \vee \underbrace{7^x = -3}_{\text{unerfüllbar für } x \in \mathbb{R}}$

$x = \log_7 5$

$x = \frac{\ln 5}{\ln 7}$

$x \approx 0{,}827$

8 h) $11^{2x} + 3 \cdot 11^x = 4$

Substitution: $u := 11^x$

$u^2 + 3u - 4 = 0$

$(u-1)(u+4) = 0$

$u = 1 \vee u = -4$

Rücksubstitution: $11^x = 1 \vee \underbrace{11^x = -4}_{\text{unerfüllbar für } x \in \mathbb{R}}$

$x = 0$

9.0 $p(x) = -\frac{1}{2}x^2 - 2x + 2{,}5$; $D_p = \mathbb{R}$; $g_m: y = mx + 3$; $x \in \mathbb{R}, m \in \mathbb{R}$.

9.1 $p(x) = -\frac{1}{2}x^2 - 2x + 2{,}5 = -\frac{1}{2}\left(x^2 + 4x + 4\right) + 2 + 2{,}5 = -\frac{1}{2}(x+2)^2 + 4{,}5$

$S(-2/4{,}5)$ $W_p = \left]-\infty; 4{,}5\right]$

9.2

x	- 7	- 6	- 5	- 4	- 3	- 2	- 1	0	1	2	3
p(x)	- 8	- 3,5	0	2,5	4	4,5	4	2,5	0	- 3,5	- 8

9.3 $G_p \cap g_m$: $\quad -\frac{1}{2}x^2 - 2x + 2{,}5 = mx + 3$

$$\frac{1}{2}x^2 + (m+2)x + 0{,}5 = 0 \quad (*)$$

Diskriminante der quadratischen Gleichung (*):

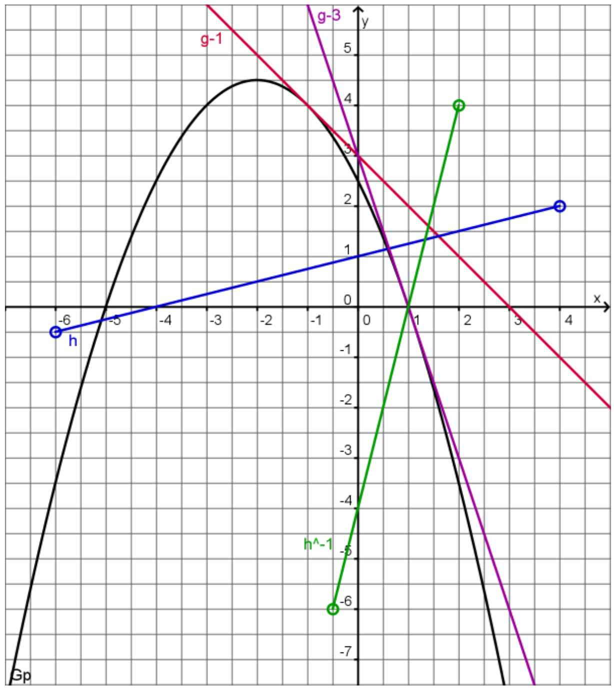
$$D = (m+2)^2 - 4 \cdot \frac{1}{2} \cdot 0{,}5 = (m+2)^2 - 1$$

$G_p \cap g_m = \{\ \}$, d. h. G_p und g_m besitzen keinen gemeinsamen Punkt

$$\Leftrightarrow \quad D < 0 \quad \Leftrightarrow \quad (m+2)^2 - 1 < 0$$

$$\Leftrightarrow \quad (m+2)^2 < 1 \quad \Leftrightarrow \quad |m+2| < 1$$

$$\Leftrightarrow \quad -1 < m + 2 < 1 \Leftrightarrow \quad \underline{\underline{-3 < m < -1}}$$



9.4 Siehe Aufgabe 9.2.

10.0 $h: x \mapsto ax + b$ mit $a, b \in \mathbb{R}$; $\quad D_h = [-6; 4[\quad ; \quad P(-4/0) \in G_h$; $Q(3/\frac{7}{4}) \in G_h$.

10.1 $P(-4/0) \in G_h \quad \Leftrightarrow \quad h(-4) = 0 \quad \Leftrightarrow \quad -4a + b = 0 \quad (I)$

$Q(3/\frac{7}{4}) \in G_h \quad \Leftrightarrow \quad h(3) = \frac{7}{4} \quad \Leftrightarrow \quad 3a + b = \frac{7}{4} \quad (II)$

$(I) - (II)$: $\quad -7a = -\frac{7}{4} \quad \Rightarrow \quad \underline{\underline{a = \frac{1}{4}}}$

$a = \frac{1}{4}$ in (I): $\quad -1 + b = 0 \quad \Rightarrow \quad \underline{\underline{b = 1}}$

10.2 $h: y = \frac{1}{4}x + 1$

$$\frac{1}{4}x = y - 1$$

$$x = 4y - 4$$

$\underline{\underline{h^{-1}: y = 4x - 4}} \qquad \underline{\underline{D_{h^{-1}} = W_h = [-0{,}5; 2[}} \qquad \underline{\underline{W_{h^{-1}} = D_h = [-6; 4[}}$

10.3 Siehe Aufgabe 9.2

11.0 $p(x) = \frac{1}{3}x^2 - 2x + 1 \quad ; \quad D_p = \mathbb{R} \quad ; \quad g_a: y = ax - 2 \quad ; \quad x \in \mathbb{R}, a \in \mathbb{R}$.

11.1 $\frac{1}{3}x^2 - 2x + 1 = 0 \quad | \cdot 3$

$$x^2 - 6x + 3 = 0$$

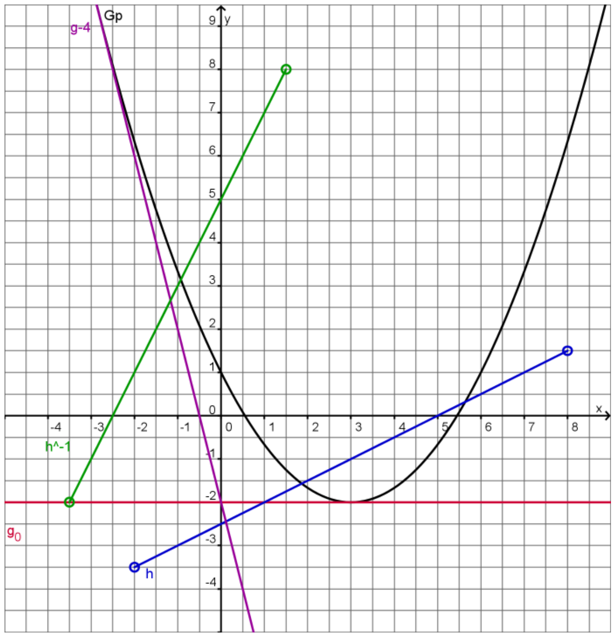
$$x_{1/2} = \frac{6 \pm \sqrt{36 - 12}}{2} = \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm 2\sqrt{6}}{2}$$

$\underline{\underline{x_1 = 3 + \sqrt{6} \approx 5{,}45}} \qquad \underline{\underline{x_2 = 3 - \sqrt{6} \approx 0{,}55}}$

11.2 $\left.\begin{array}{l} x_S = \frac{x_1 + x_2}{2} = \frac{3+\sqrt{6}+3-\sqrt{6}}{2} = 3 \\ y_S = p(3) = -2 \end{array}\right\} \Rightarrow \underline{\underline{S(3/-2)}}$ $\qquad \underline{\underline{W_p = [-2; +\infty[}}$

11.3

x	- 2	- 1	0	1	2	3	4	5	6	7	8
p(x)	6,33	3,33	1	- 0,67	- 1,67	- 2	- 1,67	- 0,67	1	3,33	6,33



11.4 $G_p \cap g_a: \quad \frac{1}{3}x^2 - 2x + 1 = ax - 2$

$\frac{1}{3}x^2 - (a+2)x + 3 = 0 \quad (*)$

Diskriminante der quadratischen Gleichung (*):

$D = [-(a+2)]^2 - 4 \cdot \frac{1}{3} \cdot 3 = (a+2)^2 - 4$

G_p und g_a besitzen zwei verschiedene Schnittpunkte $\Leftrightarrow D > 0 \Leftrightarrow (a+2)^2 - 4 > 0$

$\Leftrightarrow (a+2)^2 > 4 \Leftrightarrow |a+2| > 2$

$\Leftrightarrow a + 2 < -2 \vee a + 2 > 2$

$\Leftrightarrow \underline{\underline{a < -4 \vee a > 0}}$

11.5 Siehe Aufgabe 11.3.

12.0 $\ell: x \mapsto mx + t$ mit $m, t \in \mathbb{R}$; $D_\ell =]-2; 8]$;
$A(-1/-3) \in G_\ell$; $B(4/-0{,}5) \in G_\ell$.

12.1 $A(-1/-3) \in G_\ell \Leftrightarrow \ell(-1) = -3 \Leftrightarrow -m + t = -3 \quad (I)$
$B(4/-0{,}5) \in G_\ell \Leftrightarrow \ell(4) = -0{,}5 \Leftrightarrow 4m + t = -0{,}5 \quad (II)$

$(II) - (I): \quad 5m = 2{,}5 \Rightarrow \underline{\underline{m = 0{,}5}}$

$m = 0{,}5$ in (I): $\quad -0{,}5 + t = -3 \Rightarrow \underline{\underline{t = -2{,}5}}$

12.2 $\ell: y = 0{,}5x - 2{,}5$

$0{,}5x = y + 2{,}5$

$x = 2y + 5$

$\underline{\underline{\ell^{-1}: y = 2x + 5}}$ $\qquad \underline{\underline{D_{\ell^{-1}} = W_\ell = [-3{,}5; 1{,}5[}}$ $\qquad \underline{\underline{W_{\ell^{-1}} = D_\ell = [-2; 8[}}$

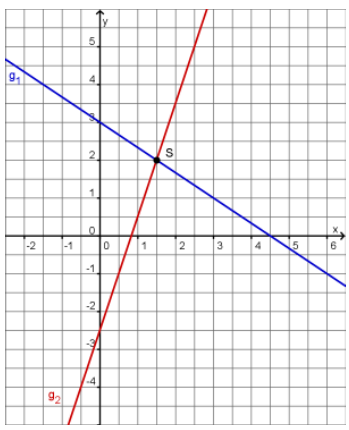
12.3 Siehe Aufgabe 11.3

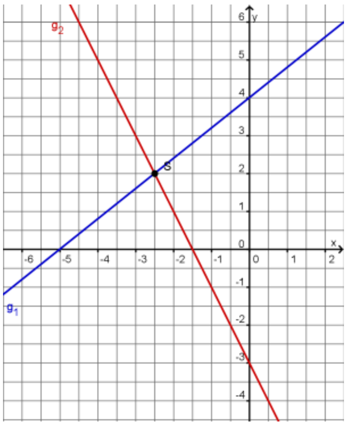
13 I $\quad 2x + 3y = 9 \Leftrightarrow y = -\frac{2}{3}x + 3 \ \hat{=}\ $ Gerade g_1

II $\quad 6x - 2y = 5 \Leftrightarrow y = 3x - 2{,}5 \ \hat{=}\ $ Gerade g_2

$g_1 \cap g_2 = \{S\}$, wobei $S(1{,}5|2)$

$\Rightarrow \underline{\underline{L = \{(1{,}5; 2)\}}}$





14 $\quad$ I $\quad 4x - 5y = -20 \quad \Leftrightarrow \quad y = \frac{4}{5}x + 4 \quad \hat{=} \quad$ Gerade g_1

II $\quad 2x + y = -3 \quad \Leftrightarrow \quad y = -2x - 3 \quad \hat{=} \quad$ Gerade g_2

$g_1 \cap g_2 = \{S\}$, wobei $S(-2{,}5; 2)$

$\Rightarrow \quad \underline{\underline{L = \{(-2{,}5; 2)\}}}$

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$$\begin{array}{lr} \text{I} & x - 2y + 5z = 19 \\ \text{II} & 3x - 4y - 5z = -5 \\ \text{III} & 4x + 2y + z = 9 \end{array}$$

$\text{I} + \text{II} \quad 4x - 6y = 14 \quad (A)$

$\text{II} + 5\,\text{III} \quad 23x + 6y = 40 \quad (B)$

$(A) + (B): \; 27x = 54$

$\underline{x = 2}$

$x = 2$ in (A): $8 - 6y = 14 \Rightarrow -6y = 6 \Rightarrow \underline{y = -1}$

$x = 2, y = -1$ in III: $8 - 2 + z = 9 \Rightarrow \underline{z = 3}$

$\Rightarrow \underline{\underline{L = \{(2; -1; 3)\}}}$

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$$\begin{array}{lr} \text{I} & 3x - 2y + 3z = -3 \\ \text{II} & x + 4y - z = 5 \\ \text{III} & -4x + 3y - 2z = 2 \end{array}$$

$\text{I} - 3\,\text{III} \quad -14y + 6z = -18 \quad (A)$

$2\,\text{II} + \text{III} \quad 11y - 4z = 12 \quad (B)$

$2(A) + 3(B): \; 5y = 0$

$\underline{y = 0}$

$y = =$ in (A): $0 + 6z = -18 \Rightarrow \underline{z = -3}$

$y = 0, z = -3$ in II: $x + 0 + 3 = 5 \Rightarrow \underline{x = 2}$

$\Rightarrow \underline{\underline{L = \{(2; 0; -3)\}}}$

17.0 $\quad f(x) = \dfrac{x^5 + 4x^3 - 7x}{x^3 - 6x} \quad ; \quad D_f \subset \mathbb{R}.$

17.1 $\quad x^3 - 6x = 0 \Leftrightarrow x(x^2 - 6) = 0 \Leftrightarrow x = 0 \vee x^2 - 6 = 0 \Leftrightarrow x = 0 \vee x = \pm\sqrt{6}$

$D_f = \mathbb{R} \setminus \{-\sqrt{6}; 0; \sqrt{6}\}$

17.2 $\quad D_f$ ist offensichtlich symmetrisch zu 0.

$$f(-x) = \frac{(-x)^5 + 4(-x)^3 - 7(-x)}{(-x)^3 - 6(-x)} = \frac{-x^5 - 4x^3 + 7x}{-x^3 + 6x} = \frac{-(x^5 + 4x^3 - 7x)}{-(x^3 - 6x)} = \frac{x^5 + 4x^3 - 7x}{x^3 - 6x} = f(x) \text{ für alle } x \in \mathbb{R}$$

$\Rightarrow G_f$ ist achsensymmetrisch zur y-Achse des Koordinatensystems.